The above analysis demonstrated the dynamics of $\alpha(t)$ and made it possible to obtain its asymptotic value as well as the dependence of the latter on the parameters of the composite. When calculating temperature fields in layered (reinforced) media within the framework of the two-temperature approach one must assess the effect of the unsteady nature of α on the accuracy of the calculation and take it into account, if necessary. A similar analysis should also be carried out for reinforced composites.

NOTATION

Here z and x denote the space variables; t is the time; $T_i(x, z, t)$ and $\hat{T}_i(z, t)$ are the temperature of the i-th component and its average cross-sectional value; q_{ij} is the density of the thermal flux from the i-th to the j-th component; α is the coefficient of heat transfer between components; C_i, λ_{xi}, and λ_{zi} are the coefficients of volumetric heat transfer and the radial and axial thermal conductivity, respectively; R_T is the coefficient of contact thermal resistance between layers; ℓ_i is the half-thickness of the layers; q_0 is the thermal flux density of the laser radiation; t_p is the length of the laser burst; p and n are the parameters of the Laplace and Fourier cosine transforms; and ℓ_k is the characteristic size of microinhomogeneities.

LITERATURE CITED

1. R. I. Nigmatulin, Dynamics of Multiphase Media. Part 1 [in Russian], Moscow (1987).
2. L. P. Khoroshun and N. S. Soltanov, Thermoelasticity of Binary Mixtures [in Russian], Kiev (1984).
3. A. A. Nayfeh, Trans. ASME: J. Appl. Mech., 42, No. 2, 399 (1975).
4. G. C. Bart, C. J. Hoogendoorn, and P. B. J. Schaareman, Warme- und Stoffubertragung, 20, No. 4, 269 (1986).
5. A. V. Lykov, Theory of Thermal Conductivity [in Russian], Moscow (1967).
6. R. L. Shoemaker, High Temperatures—High Pressures, 18, No. 6, 645 (1986).
7. I. V. Goncharov and V. L. Mikov, "Temperature field of a reinforced composite under boundary conditions of the second kind," Dep. VIMI No. D07661, January 19, 1988.
8. I. V. Goncharov and V. L. Mikov, Inzh.-Fiz. Zh., 58, No. 2, 311 (1990).

PROBLEM OF HEAT AND MASS TRANSFER DURING SHORT-TIME PHASE CONTACT

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The associated mixed boundary-value problem of multicomponent mutually related heat and mass transfer during short-time contact of two phases with arbitary dimensionalities of the transfer vector potentials in them through a boundary with selective penetrability during excitation of material flows in each of the phases, which are absent in the other phase, is formulated and solved. This is done with the purpose of generalizing the model of phase penetration and restoration in the theory of mass exchange, and of similar models in the theory of heat exchange, based on the phenomenon of short-time contact interaction. The validity limits of these models are estimated. An effect is observed of internal phase flows on the intensity of nonstationary interphase exchange.

The contemporary intense development of material processing technology leads to an enhanced role of nonstationary mutually related exchange processes in comparison with the stationary decoupled ones. This fact is so far not sufficiently reflected in the solution of problems of heat and mass transfer (HMT) at small Fourier numbers, for short-time contact (SC) phases. The physical model concepts have been developed well for both heat- and mass-transfer, but separately. In the theory of mass exchange they are represented by permeation (Higby) and phase

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restoration models, while in heat exchange calculations one uses the theory of fluidization, simplified models of gushing layers, and models of conductive drying. The corresponding mathematical problems have simple analytic solutions (see the review below of special solutions of problems (1)-(5)). It is useful to clarify whether this simplicity is conserved in more general statements of SC problems, when the mutual effects of HMT and the selective permeability of the surface contact are included.

Consider the case of SC phases with possible selective permeability of surface contact. Different systems of equations (SE) of HMT with different dimensionality of the vector Z of transport potentials may be valid in the phases. At this phase we will not specify the meaning of the components of the vector Z and will not restrict its dimensionality, possibly larger than three [1]. The low penetration depth of the perturbation fields of the transport potentials in the SC models makes it possible to neglect the finite sizes of contacting phases (first and second) and the curvature of the contact surface. The distribution of transport potentials is assumed uniform in each phase up to contact. Far from the contact planes the transport potential fields are not perturbed. There exist no concentrated sources in the phases, and the distributed power is expressed in terms of derivatives of Z, as was done in studies of A. V. Lykov in the analysis of MHT in capillary-porous bodies. All parameters are assumed to be constant. According to these assumptions for each phase we have conservation equations of transport substance in local form in the absence of convective transport

$$C_i \frac{\partial Z_i}{\partial \tau} = L_i \frac{\partial^2 Z_i}{\partial z^2}, \quad i = 1,\ 2, \tag{1}$$

initial conditions

$$Z_i(z,\ 0) = Z_{i0}, \quad \forall z \neq 0, \quad i = 1,\ 2, \tag{2}$$

and conditions at infinite distance from the contact surface

$$Z_i((-1)^i \infty,\ \tau) = Z_{i0}, \quad \forall \tau > 0, \quad i = 1,\ 2, \tag{3}$$

where the elements C_i are defined as ratios of stored amount of substance to the corresponding transport potential, and depend on the selected units of measuring Z_i, as well as the elements L_i, while the dimensionalities of the vectors Z_i are equal to n_i and do not generally coincide. For definiteness let $n_2 \geq n_1$, i.e., the number of mutually transported substances in the second phase is larger than in the first phase, for example, thermal moisture transfer in a capillary-porous body (phase 2) in contact with a metallic hot body (phase 1).

Initially we consider for simplicity the case in which all components $Z_1 \in R_1$ are represented in $Z_2 \in R_2$ — we have the insertion $R_1 \subset R_2$. Obviously, the boundary $z = 0$ is impenetrable in this case for the $n_2 - n_1$ components of Z_2 in $R_2 \backslash R_1$. We introduce the reduction operation of a vector or matrix from R_2 into R_1 ($^-$) and the extension operation by zeros from R_1 into R_2 ($^=$). The equilibrium condition at the boundary is then written in the form of a condition of the first kind

$$\bar{Z}_2(0,\ \tau) = K_\Gamma Z_1(0,\ \tau) \in R_1 \ \forall \tau > 0, \tag{4}$$

where K_Γ is a diagonal matrix of $n_1 \times n_1$ equilibrium constants. In particular, if the topic of discussion is multicomponent diffusion and heat propagation, the diagonal of K consists of Henry constants or coefficients of the linearized representation of equilibrium phase compositions [2] and unities, corresponding to equilibrium heat at certain phase temperatures in identical measurement units. Assuming absence of concentrated transport resistance at the phase separation boundary, we write the flow balance at the boundary in the form

$$\overline{\overline{L_1 \frac{\partial Z_1(0,\ \tau)}{\partial z}}} - L_2 \frac{\partial Z_2(0,\ \tau)}{\partial z} = 0 \in R_2, \quad \forall \tau > 0, \tag{5}$$

being a condition of the second kind. Along with the matching conditions (4), (5), the boundary-value problem (1)-(5) must be solved in the regions $z > 0$ and $z < 0$ for $\tau > 0$.

The solution of the problem stated is known for $n_1 = n_2 = 1$ in the case of purely thermal contact. The solution is carried out in [3] by the method of integral transforms for the first boundary-value problem (1) in the half-space $z > 0$ for arbitrary n_2, where one uses implicitly the positive property of the eigenvalues $C^{-1}L$. Transport effects in the first phase and selective permeability were not considered in this case. The mixed boundary-value problem for (1) in the region $z > 0$ was solved in [4] for $n_2 = 2$ and in [5] for $n_2 = 3$, in which case the positivity of eigenvalues was proved in [4] under specific conditions of HMT in a capillary-porous body, and the selective permeability of the contact surface was accounted for. To simplify the calculations we assume in the following satisfaction of the positivity condition of eigenvalues for (1) and the presence of a basis of eigenvectors of $C^{-1}L$. By definition the matrix T_i of eigenvectors of the matrix $C_i^{-1}L_i$, corresponding to its n_i eigenvalues and combined in the diagonal matrix ν_i^2, is such that

$$\nu_i^2 = T_i^{-1}C_i^{-1}L_iT_i, \quad i = 1,\ 2. \tag{6}$$

The basis of obtaining a simple solution in [4, 5] was the Boltzmann substitution [6]. For (1)-(5) it must be carried out in both regions:

$$\xi_i = (-1)^i \frac{z}{2\tau^{0,5}}, \quad i = 1,\ 2.$$

We also introduce $K_2 = K_\text{г} + I_2 - I_1$, and carry out the replacement of variables:

$$Z_2 = K_2Z_2^*, \quad C_2^* = C_2K_2, \quad L_2^* = L_2K_2. \tag{7}$$

The eigenvalues of $C_2^{*-1}L_2^*$ coincide with those of $C_2^{-1}L_2$, and the eigenvectors form the matrix $T_2^* = K_2^{-1}T_2$. We write down the problem (1)-(5) in the new variables, omitting the superscript * for the second phase. From (1) we have

$$2\xi_iZ_i' + C_i^{-1}L_iZ_i'' = 0 \in R_i, \quad i = 1,\ 2, \tag{8}$$

where the prime denotes differentiation with respect to ξ. Conditions (2) and (3) are combined into

$$Z_i(\infty) = Z_{i0},\ i = 1,\ 2, \tag{9}$$

the flow balance (5) transforms to the form

$$\overline{\overline{L_1Z_1'(0)}} + L_2Z_2'(0) = 0 \in R_2, \tag{10}$$

and the equilibrium condition (4) transforms to the continuity condition of the transport potentials during transition through the boundary

$$Z_1(0) - \bar{Z}_2(0) = 0 \in R_1. \tag{11}$$

The variable replacement $\xi_i = \zeta_i^{0,5}$, $Z_i' = P$ transforms (8) to an autonomous system of $n_1 + n_2$ first-order linear ordinary differential equations in P_i with checkered matrices, being the direct sum [7] of $-L_1^{-1}C_1$ and $-L_2^{-1}C_2$. This makes it possible to indicate the structure of the required solutions in the form of a linear combination of supplementary error integrals erfc (ξ_i/ν_{ij}), where ν_{ij} is the square root of the j-th eigenvalue of $C_i^{-1}L_i$, as well as in [4, 5]. We seek a solution of (8)-(11) in the form

$$Z_i(\xi_i) = Z_{i0} + T_i \operatorname{erfc}(\nu_i^{-1}\xi_i)A_i, \quad i = 1,\ 2, \tag{12}$$

where the operation erfc is applied to the diagonal elements of the matrix $\nu_i^{-1}\xi_i$ element by element:

$$Z_i(\xi_i) = Z_{i0} + A_{i1}\operatorname{erfc}(\xi_i/\nu_{i1})\,t_{i1} + \ldots + A_{in}\operatorname{erfc}(\xi_i/\nu_{in_i})\,t_{in_i},$$

where the j-th eigenvector t_{ij} is the solution of SE $C_i^{-1}L_i t_{ij}=\nu_{ij}t_{ij}$. It is easily seen that the system of equations (8) and conditions (9) are satisfied identically while the vectors $A_1 \in R_1$ and $A_2 \in R_2$ of integration constants are subject to conditions (10) and (11).

In analyzing "pure" heat transfer in a single phase it is useful to introduce the heat accomodation (thermal activity) coefficient [8] $\kappa = c\rho[\lambda/c\rho]^{0,5}$, determining the phase activity contribution to heat transfer, and unrelated to the time constant τ_k (with the hydrodynamic environment). The heat transfer coefficient is in this case $2\kappa/[\pi\tau_k]^{0,5}$ [8]. In mass exchange theory, according to the Higby models and the restored phases the mass transfer coefficient is represented in the form $\beta = 2D^{0,5}/[\pi\tau_k]^{0,5}$, where D is the diffusion coefficient. The square root of the diffusion coefficient is a convenient parameter in calculating diffusion in chemical kinetics [9], in solid alloys [10], and in other cases. This quantity has not acquired any name in the literature, though well investigated and discussed is the power $m = \partial \ln\beta/\partial \ln D$, equal to $\frac{1}{2}$ according to Higby theorems and phase restoration, and 1 according to film theory. In the problem of multicomponent mutually related HMT with SC we introduce similar characteristic phase properties — the potential accomodation matrices

$$\Lambda_i = C_i T_i \nu_i T_i^{-1} = L_i T_i \nu_i^{-1} T_i^{-1}, \quad i = 1,\ 2, \tag{13}$$

and form from them the contact matrix

$$\Lambda = \overline{\overline{\Lambda}}_1 (\overline{\overline{\Lambda}}_1 + \Lambda_2)^{-1} \Lambda_2 = \Lambda_2 (\overline{\overline{\Lambda}}_1 + \Lambda_2)^{-1} \overline{\overline{\Lambda}}_1, \tag{14}$$

determining the basic properties of SC of the given phases, and allowing one to write down a solution of problem (8)-(11) in the simplest form.

We note that $\det \Lambda_i > 0$, where Λ is the vanishing operator on $R_2 \setminus R_1$, and, moreover, $\overline{\overline{\Lambda}} = \Lambda$, i.e., nonvanishing elements of Λ occur only in rows and columns corresponding to R_1. For $n_2 = n_1$, when it makes sense to formulate additivity rules of transport resistance [2], this rule is valid by the definition of (14):

$$\Lambda^{-1} = \Lambda_1^{-1} + \Lambda_2^{-1}.$$

When the minimum eigenvalue of one of the accomodation potential matrices Λ_i is much larger than the maximum eigenvalue of another Λ_j, $\Lambda \to \Lambda_j$. This is similar to the equality between the mass transfer (heat transfer) coefficient and mass release (heat release) coefficient during resistance-limited transport by one of the phases.

The potential accomodation matrices can be written in a form similar to the heat accomodation coefficient: $\Lambda_i = C_i[C_i^{-1}L_i]^{0,5}$, using the definition of the square root matrix [11].

To search the vector constants A_1 and A_2 in the solution (12) we introduce the notation $\Delta Z = \overline{Z}_{20} - Z_{10} \in R_1$, and write conditions (10), (11), with account of (12), (13), in the form

$$\overline{\overline{\Lambda_1 T_1 A_1}} + \Lambda_2 T_2 A_2 = 0 \in R_2, \tag{15}$$

$$T_1 A_1 = \overline{T_2 A_2} + \Delta Z \in R_1. \tag{16}$$

Substituting (16) into (15), we obtain

$$\overline{\overline{\Lambda_1 \overline{T_2 A_2}}} + \Lambda_2 T_2 A_2 = -\overline{\overline{\Lambda_1 \Delta Z}} \in R_2. \tag{17}$$

We note that $\forall x \in R_2$:

$$\overline{\overline{\Lambda_1 \overline{x}}} = \overline{\overline{\Lambda}}_1 \overline{\overline{\overline{x}}} = \overline{\overline{\Lambda}}_1 [\overline{x} + (x - \overline{x})] = \overline{\overline{\Lambda}}_1 x,$$

including both $x = T_2A_2$ and $x = \overline{\overline{\Delta Z}}$. Consequently, (17) is equivalent to

$$(\overline{\overline{\Lambda}}_1 + \Lambda_2) T_2 A_2 = -\overline{\overline{\Lambda}}_1 \Delta \overline{\overline{Z}} \in R_2.$$

Multiplying on the left by $(\overline{\overline{\Lambda}}_1 + \Lambda_2)^{-1}$, this equality can be transformed to the form

$$T_2A_2 = -(\overline{\overline{\Lambda}}_1 + \Lambda_2)^{-1}\overline{\overline{\Lambda}}_1\Delta\overline{\overline{Z}} \in R_2 \tag{18}$$

or, with account of (14) and the nondegeneracy of Λ_2:

$$A_2 = -T_2^{-1}\Lambda_2^{-1}\Lambda\overline{\overline{\Delta Z}} \in R_2. \tag{19}$$

To search A_1 we substitute (18) into (16):

$$T_1A_1 = \Delta Z - \overline{(\overline{\overline{\Lambda}}_1 + \Lambda_2)^{-1}\overline{\overline{\Lambda}}_1\overline{\overline{\Delta Z}}} \in R_1.$$

We note here that for the whole matrix M over R_2 and for the whole vector $x \in R_1$

$$\overline{M\overline{\overline{x}}} = \overline{M}x,$$

and we carry out the transformations:

$$T_1A_1 = \Delta Z - \overline{(\overline{\overline{\Lambda}}_1 + \Lambda_2)^{-1}\overline{\overline{\Lambda}}_1}\Delta Z = [I_1 - \overline{(\overline{\overline{\Lambda}}_1 + \Lambda_2)^{-1}\overline{\overline{\Lambda}}_1}]\,\Delta Z =$$

$$= \overline{[I_2 - (\overline{\overline{\Lambda}}_1 + \Lambda_2)^{-1}\overline{\overline{\Lambda}}_1]}\,\Delta Z = \overline{(\overline{\overline{\Lambda}}_1 + \Lambda_2)^{-1}[\overline{\overline{\Lambda}}_1 + \Lambda_2 - \overline{\overline{\Lambda}}_1]}\,\Delta Z =$$

$$= \overline{(\overline{\overline{\Lambda}}_1 + \Lambda_2)^{-1}\Lambda_2}\Delta Z = \Lambda_1^{-1}\Lambda_1\,\overline{(\overline{\overline{\Lambda}}_1 + \Lambda_2)\,\Lambda_2}\Delta Z.$$

For any two matrices, M_1 over R_1 and M_2 over R_2, $M_1\overline{M}_2 = \overline{M_1M_2}$, therefore

$$T_1A_1 = \Lambda_1^{-1}\overline{\Lambda}\,\Delta Z \in R_1,$$

whence

$$A_1 = T_1^{-1}\Lambda_1^{-1}\overline{\Lambda}\,\Delta Z \in R_1. \tag{20}$$

Thus, the problem (8)-(11) has been solved.

In the HMT problem under consideration there exists a confinement through the interphase boundary, which can be substantial in several cases, the enclosure $R_1 \subset R_2$. This implies that in the first phase there is no transfer of materials which are not transferred into the second phase. Within the solution obtained by formally introducing into R_2 defect potentials which exist in the first phase, this defect should not be adjusted, since the separation boundary is impenetrable for the corresponding fluxes. An example could be thermal contact of a capillary-porous body through a thin metallic object with a heat carrier containing impurities. The corresponding general problem renders the calculations quite unwieldy, and it is not possible to obtain a constructive solution of the system (15), (16). However, writing down a solution by analogy has shown its validity by direct verification. We provide results of solving this more general problem of mutual HMT.

Equations (1)-(3), (6)-(9), (12), (13) remain unchanged. The insertion $R_1 \subset R_2$ is not assumed. The spaces $R_{1+2} = R_1 \cup R_2$, $R_{12} = R_1 \cap R_2$ are introduced, and the operations of extending a vector or a matrix into R_{1+2} by zeros (=) and reduction into R_{12} (-), generalizing the operations introduced earlier. We put $K_2 = K_\Gamma + I_2 - I_{12}$ in (7). The flow balance condition (10) becomes

$$L_1Z_1'(0) + L_2Z_2'(0) = 0 \in R_{1+2}, \tag{21}$$

and the same potential continuity condition (similar to (11)) acts only in R_{12}:

$$\overline{Z}_1(0) - \overline{Z}_2(0) = 0 \in R_{12}. \tag{22}$$

The definition (14) is generalized as follows:

$$\Lambda = \bar{\bar{\Lambda}}_1 (\bar{\bar{\Lambda}}_1 + \bar{\bar{\Lambda}}_2)^{-1} \bar{\bar{\Lambda}}_2 = \bar{\bar{\Lambda}}_2 (\bar{\bar{\Lambda}}_1 + \bar{\bar{\Lambda}}_2)^{-1} \bar{\bar{\Lambda}}_1.$$

Here, as well as in the previous problem, det $\Lambda_i > 0$, where Λ is the vanishing operator over $R_{1+2} \backslash R_{12}$ $\bar{\bar{\Lambda}} = \Lambda$. The solution of the problem (8), (9), (21), (22) in the form (12) has the coefficients

$$A_1 = T_1^{-1} (\bar{\bar{\Lambda}}_1 + \bar{\bar{\Lambda}}_2)^{-1} \bar{\bar{\Lambda}}_2 \Delta Z \in R_1, \quad A_2 = -T_2^{-1} (\bar{\bar{\Lambda}}_1 + \bar{\bar{\Lambda}}_2)^{-1} \bar{\bar{\Lambda}}_1 \Delta Z \in R_2,$$

where $\Delta Z = \bar{Z}_{20} - \bar{Z}_{10}$. It is seen that the solution obtained above (19), (20) is a special case of the latter expressions.

Quite important is the issue of validity limits of the SC model. Not dwelling on the validity limits of all remaining assumptions, we consider the basic problem whether the contact can be assumed to be short-time. Despite the wide range of applications of this model, the unique condition characterizing its validity remains the restriction formulated by O. Krischer [12]

$$\mathrm{Fo} = \frac{a\tau_{\mathrm{K}}}{l^2} < 0{,}106. \tag{23}$$

Here a is the temperature conductivity coefficient of a planar film of thickness ℓ with initial temperature T_0, one of whose surfaces ($z = 0$) is subject to temperature $T_c \neq T_0$ during the time of contact. Condition (23) guarantees that the temperature variation at the other film surface ($z = \ell$) does not exceed $|T_c - T_0|$ by more than 3%. The generalization of (23) to the corresponding case of multicomponent HMT in the film is the same condition, in which for a one must use the largest eigenvalue λ_{max} of the matrix $C^{-1}L$. Depending on the purpose of study, the level of allowed oscillations of the various Z components at the exterior boundary $z = \ell$ can differ, therefore either attenuation or enhancement of condition (23) is possible or a combination of this type of conditions. In specific cases these estimates are carried out quite simply by using solutions of type (12). The same approach can be obviously extended to the case of contact of two films of finite thickness.

However, the occurrence of these conditions, guaranteeing calculation accuracy of potential fields in a film of finite sizes, has drawbacks. It is unrelated to the basic problem of the HMT calculation, the determination of exchange process intensities, and therefore is irrelevant to the calculation accuracy of material transfer fluxes. Besides, in real situations a restriction related to the curvature of the contact surface (granular surface, gas bubbles, cylindrical objects, etc.), but not to the finite size of the contacting bodies, can be active. Detailed analysis of the effect of finite size and curvature on the flow calculation accuracy was carried out in [13] for the case dim Z = 1, when one of the phases has infinitely large capacity, while the second occupies a canonical region of space (the film interior of thickness 2ℓ, cylinder and sphere of radius ℓ).

A substantial restriction can also be the presence of angles at the violating surface. To estimate the validity limits of the SC model in this case one can use the solution obtained in [14] of the first boundary-value problem for the thermal conduction equation inside an infinite wedge with opening angle $\Theta = \pi/k$, where $k = 3, 5, 7, \ldots,$. Calculating the time-averaged thermal flux contact through two surfaces of length ℓ, treated asymptotically for $k \to \infty$ and subsequent approximations, provides the possibility of determining the required error. We use the solution [15] in estimating heat release from a cylinder of circular cross section and a sphere in the external region.

In Fig. 1 we construct the calculated dependences of the ratio of the heat release coefficient to its value calculated for a planar boundary half-space according to Krischer [12].

Turning to multicomponent transport, for a rough estimate one can also substitute λ_{max} for a. In treating HMT in two contacting phases one requires more specific analysis, but here too λ_{max} plays a decisive role.

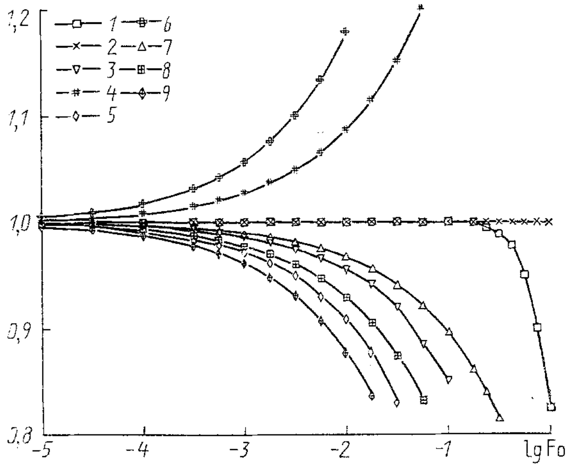

Fig. 1. The ratio of the calculated heat release coefficient to its value according to O. Krischer as a function of the Fourier number during heat release: 1) in the interior of an infinite film; 2) in the exterior of an infinite film; 3) in the interior of an infinite cylinder; 4) in the exterior of an infinite cylinder; 5) in the interior of a sphere; 6) in the exterior of a sphere; 7, 8, 9) in the interior of an infinite wedge with opening angle 150°, 120°, and 90°, respectively.

To calculate the flows we note that in both solutions obtained

$$L_1 Z_1'(0) = -L_2 Z_2'(0) = -\frac{2}{\pi^{0,5}} \Lambda \Delta Z,$$

and, transforming from ξ_i to the original variables z and τ, we find the vector J of instantaneous flow densities from the first to the second phase

$$J(\tau) = \frac{1}{[\pi\tau]^{0,5}} \Lambda (\bar{\bar{Z}}_{10} - \bar{\bar{Z}}_{20}). \tag{24}$$

The flux surface density averaged over the contact time is

$$J_{av}(\tau_K) = \frac{2}{[\pi\tau_K]^{0,5}} \Lambda (\bar{\bar{Z}}_{10} - \bar{\bar{Z}}_{20}), \tag{25}$$

which fully corresponds to most computational equations of heat transfer, mass transfer, and multicomponent mass transfer [2]. The product

$$K = \frac{2\Lambda}{[\pi\tau_K]^{0,5}} \tag{26}$$

can be called the transport potential matrix for mutually related HMT during SC of two phases. The decisive role of hydrodynamic contact conditions is reflected in K by the value of the contact time τ_k, while the properties of contacting media are accounted for by the contact matrix Λ. We note that in stating and solving problems of the first kind in the region $z > 0$ the same expressions (24), (25) are valid for the flux densities, where Λ is replaced by the transport potential matrix Λ_2, and Z_{10} is the surface value of Z_2 (in which case $A_1 = 0$, $A_2 = T_2^{-1}(Z_{10} - Z_{20})$). This solution can be obtained by using a matrix cell representation by the limiting transition corresponding to an unbounded increasing capacity or a conductivity of the first phase, when $Z_1(0) \to Z_{10}$.

One more comment on the properties of the solutions obtained is appropriate. In most experimental studies of mass exchange the value of the power in the dependence of the mass release coefficient on the diffusion coefficient m is larger than $\frac{1}{2}$, usually in the interval 0.5-0.75 [16]. As a rule, values $m > \frac{1}{2}$ are interpreted in favor of film theory, though, possibly, this is an argument in favor of HMT relevance. We illustrate this point on a simple example, when the second phase limits transport, and the boundary is permeable only to one component, $n_1 = 1$, $n_2 = 2$. Omitting for brevity the phase number (=2), putting $C_2^{-1}L_2 = A$, and indexing the matrix elements by row and column numbers, we have

$$T_2 = \begin{pmatrix} (\nu_1^2 - A_{22})/A_{21} & (\nu_2^2 - A_{22})/A_{21} \\ 1 & 1 \end{pmatrix}, \qquad T_2^{-1} = \frac{A_{21}}{\nu_1^2 - \nu_2^2} \begin{pmatrix} 1 & -(\nu_2^2 - A_{22})/A_{21} \\ -1 & (\nu_1^2 - A_{22})/A_{21} \end{pmatrix},$$

$$\Lambda_2 = \frac{1}{\nu_1 + \nu_2} \begin{pmatrix} L_{11} + \nu_1\nu_2 C_{11} & L_{12} \\ L_{21} & L_{22} + \nu_1\nu_2 C_{22} \end{pmatrix},$$

$$\Lambda = \frac{1}{\nu_1 + \nu_2} \begin{pmatrix} L_{11} + \nu_1\nu_2 C_{11} - L_{12}L_{21}/(L_{22} + \nu_1\nu_2 C_{22}) & 0 \\ 0 & 0 \end{pmatrix}.$$

According to (26) the density of scalar flow through the phase boundary is proportional to Λ_{11}, while in the case of pure (unmixed) transport Λ_{11} is replaced by the product $C_{11}[C_{11}^{-1} \cdot L_{11}]^{0,5}$. The quantity χ, which may be termed the mutual transport coefficient, is:

$$\chi = \frac{\Lambda_{11}}{[C_{11}L_{11}]^{0,5}} = \left[1 - \frac{L_{12}L_{21}}{L_{11}L_{22}} \frac{1 + 2s}{1 + s}\right]^{0,5}, \tag{27}$$

where $s = [\det (A)]^{0,5}/A_{22}$.

In systems for which the Onsager relations [3] are valid the matrix L is symmetric, $L_{12}L_{21} > 0$ and $0 < \chi < 1$. L is not always symmetric, nevertheless the property $\chi < 1$ can be assumed to be quite general, since, according to (27), only the signs of L_{12} and L_{21} are generally valid for it. We determine the power:

$$m = \frac{\partial \ln \Lambda_{11}}{\partial \ln A_{11}} = \frac{1}{2} \left[1 + \frac{(1 - \chi^2) A_{11}}{(1 + s)(1 + 2s) A_{22}}\right]. \tag{28}$$

As is seen, $m > \frac{1}{2}$ for $L_{12}L_{21} > 0$. Thus, despite the permeability of the separation boundary for one material only, its rate of transport depends on the fluxes of other materials, not intersecting the boundary. The Eckerman corrections to heat flux and to the similar correction of mass flux through the film for simultaneous heat and mass transfer are well-known [16], but an effect on stationary transport is possible only during transition of additional materials from phase to phase. Under nonstationary conditions of SC phases Eqs. (27) and (28) demonstrate an effect of transport mutuality even in the absence of additional fluxes through the boundary. The additional fluxes reduce the intensity of the fundamental exchanges, though they are internal.

The noncoincidence of eigenvalues of $C^{-1}L$ used above is not a major restriction — in the opposite case Z' can be sought in the form of a linear combination of the functions $\exp(-\xi^2/\nu_i^2)$, $\xi^2 \exp(-\xi^2/\nu_i^2)$, $\xi^4 \exp(-\xi^2/\nu_i^2) \ldots$, and not only $\exp(-\xi^2/\nu_i^2)$.

When the association conditions are of more general form, more precisely, when equality (5) is written with the replacement of kinetic coefficient matrices L_1 and L_2 by arbitrary matrices Q_1 and Q_2 of corresponding dimensionalities, the problem (1)-(5) has similar properties. Introducing $G_i = Q_iL_i^{-1}$, we observe that the new problem has the same form of solution (12) with the same T_i, Λ_i, but is coefficients A_i are determined, instead of (19) and (20), by the equations

$$A_2 = -T_2^{-1} (G_2\Lambda_2)^{-1} \Lambda_G \overline{\overline{\overline{\Delta Z}}}, \quad A_1 = T_1^{-1} (G_1\Lambda_1)^{-1} \overline{\Lambda}_G \overline{\overline{\overline{\Delta Z}}},$$

in which the new contact matrix is:

$$\Lambda_G = G_2\Lambda_2 (\overline{\overline{G_1\Lambda_1}} + G_2\Lambda_2)^{-1} \overline{\overline{G_1\Lambda_1}} = \overline{\overline{G_1\Lambda_1}} (\overline{\overline{G_1\Lambda_1}} + G_2\Lambda_2)^{-1} G_2\Lambda_2.$$

In that case the expressions for the flux densities and the transfer potential matrix (24)-(26) are retained, replacing Λ by Λ_G. The case of absence of insertions $R_1 \subset R_2$ for the problem with matrices Q_i involves in these three equations the same changes as in the problem with matrices L_i.

Thus, application of short-time contact models, and the introduction of extension and reduction operations of vectors and matrices, make it possible to obtain a simple analytic solution of the quite general problem of multicomponent mutually related nonstationary HMT through a boundary with selective permeability. The method suggested of generalizing parameters convenient for calculations, such as mass exchange, heat exchange, and heat accomodation coefficients, retains the shape of the equations of interphase exchange in the case of more than one of its driving forces.

The dependence was established of exchange fluxes between phases on the internal fluxes of other substances induced by them in the phases without intersecting the contact surface. The power values exceeding $\frac{1}{2}$ in the dependence of the mass release coefficient on the diffusion coefficient, usually interpreted in favor of film theory, can be explained by the effect of internal fluxes in the phase.

Numerical values were determined for the validity limits of the hypothesis of short-time contact by the Fourier criterion, starting from a calculation accuracy of interphase fluxes.

The results obtained can be useful regarding experiments on nonstationary HMT, in analyzing exchange processes in industry, and in design problems.

NOTATION

Here T denotes the matrix of eigenvectors; Z is the transport potential vector; τ is time in sec; z is the coordinate along the normal to the contact plane with a zero at the phase separation boundary; λ_j is an eigenvalue of the matrix; ν is the matrix of square roots of eigenvalues; C is a diagonal matrix of capacity coefficients; L is a matrix of kinetic coefficients; R_i is Euclidean space of dimension n_i; I_i is the unit matrix over R_i; Fo is the Fourier number; ξ is a Boltzmann variable; Q is a matrix of boundary conditions of the second kind; c is the specific heat capacity; ρ is the density; λ is the heat conductivity; i is the phase number; and j denotes the eigenvalue number.

LITERATURE CITED

1. S. V. Nerpin and A. F. Chudnovskii, Energy and Mass Exchange in the Plant—Ground—Air System [in Russian], Leningrad (1975).
2. V. V. Kafarov and V. N. Vetokhin, Foundations of Automatized Design of Chemical Products [in Russian], Moscow (1987).
3. A. V. Lykov and Yu. A. Mikhailov, Theory of Heat and Mass Transfer, Israel Program for Scientific Translations (1965).
4. V. V. Migunov and R. A. Sadykov, Teor. Osn. Khim. Tekhnol., 19, No. 1, 103-105 (1985).
5. V. V. Migunov, R. A. Sadykov, and N. P. Sotnikova, Inzh.-fiz. Zh., 46, No. 2, 327 (1984).
6. A. V. Lykov, Theory of Heat Conductivity [in Russian], Moscow (1967).
7. G. A. Korn and T. M. Korn, Mathematical Handbook for Scientists and Engineers, McGraw-Hill, New York (1961).
8. P. G. Romankov and V. F. Frolov, Heat Exchange Processes of Chemical Technology [in Russian], Leningrad (1982).
9. D. A. Frank-Kamenetskii, Diffusion and Heat Transfer in Chemical Kinetics, Plenum Press, New York (1969).
10. S. D. Gertsriken and I. Ya. Dekhtyar, Diffusion in Metals and Alloys in the Solid Phase [in Russian], Moscow (1960).
11. R. E. Bellman, Introduction to Matrix Analysis, 2nd ed., McGraw-Hill, New York (1970).
12. O. Krischer, Scientific Foundations of Drying Technology [Russian translation], Moscow (1961).
13. V. V. Migunov, The Drying Process of Aminoacid and Antibiotic Microsynthesis Products, Thesis, Kazan' (1985).
14. V. N. Vaenskii, Inzh.-fiz. Zh., 46, No. 2, 329 (1984).
15. H. S. Carslaw and J. C. Jaeger, Conduction of Heat of Solids, Oxford (1959).
16. T. K. Sherwood, R. L. Pigford, and C. R. Wilke, Mass Transfer, McGraw-Hill, New York (1975).